Bose-Einstein correlations and color reconnection in W-pair production

S.V. Chekanov[1,a], E.A. De Wolf[2,b], W. Kittel[3]

[1] Argonne National Laboratory, 9700 S.Cass Avenue, Argonne, IL 60439, USA
[2] Department of Physics, Universitaire Instelling Antwerp, B-2610 Wilrijk, Belgium
[3] High Energy Physics Institute Nijmegen (HEFIN), University of Nijmegen/NIKHEF, NL-6525 ED Nijmegen, The Netherlands

Abstract. We propose a systematic study of Bose-Einstein correlations between identical hadrons coming from different W decays. Experimentally accessible signatures of these correlations as well as of possible color reconnection effects are discussed on the basis of two-particle inclusive densities.

1 Introduction

One of the important problems in the study of the e^+e^- annihilation at LEP2 energies is the understanding of production and decay of W-boson pairs. Due to the fact that hadrons originating from W decay overlap in space and are created in time almost simultaneously, it is natural to expect that there are correlations between hadrons originating from different W decays due to color reconnection and Bose-Einstein (BE) interference. These effects may affect the accuracy with which the W mass could be measured [1–3].

The DELPHI Collaboration has estimated both effects [4,5]. At present level of statistics, no evidence for these effects has been found. However, no systematic theoretical treatment of the BE effect in W-pair production has been given so far.

The problem of BE correlations cannot be separated from the color reconnection effect. For the color reconnection phenomenon, theoretical model investigations have recently been performed [6–9]. For example, it was proposed in [6] to measure a difference between the mean hadron multiplicity in four-jet final states ($W^+W^- \rightarrow \bar{q}_1\bar{q}_2q_3q_4$) and twice the hadronic multiplicity in two-jet events ($W^+W^- \rightarrow \bar{q}q l\bar{\nu}_l$). Having clear advantages at the present level of low statistics, this method, however, cannot be sensitive to all possible correlations which may exist due to cross-talk between hadrons and may be experimentally accessible in the near future.

In this paper we present a systematic study of both effects leading to a stochastic dependence between hadrons coming from different W decays. Our study is mainly limited to a discussion of two-particle inclusive densities but can easily be generalized to higher-order correlations.

[a] On leave from the Institute of Physics, AS of Belarus, Skaryna av.70, Minsk 220072, Belarus
[b] Onderzoeksdirecteur FWO (Vlaanderen), Belgium

2 Independent W-pair decay

2.1 Many-particle inclusive description

In this subsection we shall give a very general formalism of independent WW decay using generating functionals for many-particle inclusive densities (see [10] for a review).

A distribution of final-state particles produced in four-jet WW decay in a phase-space domain Ω is fully determined by the generating functional

$$\mathcal{R}^{\mathrm{WW}}[u(p)] = 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \int_{\Omega} \rho^{\mathrm{WW}}(p_1, p_2, \ldots, p_n)$$
$$\times u(p_1) \ldots u(p_n) \prod_{i=1}^{n} \mathrm{d}p_i, \qquad (1)$$

where $\rho^{\mathrm{WW}}(p_1, p_2, \ldots, p_n)$ is the n-particle inclusive distribution with p_i being the 4-momentum of ith particle. The inclusive densities can be recovered from the functional differentiation of (1)

$$\rho^{\mathrm{WW}}(p_1, p_2, \ldots, p_n)$$
$$= \partial^n \mathcal{R}^{\mathrm{WW}}[u(p)]/\partial u(p_1) \ldots \partial u(p_n) \mid_{u=0} . \qquad (2)$$

Since high-order inclusive densities contain redundant information from lower-order densities, it is advantageous to consider the n-particle (factorial) cumulant correlation functions $C^{\mathrm{WW}}(p_1, p_2, \ldots, p_n)$ which are obtained from the generating functional

$$\mathcal{G}^{\mathrm{WW}}[u(p)] = \ln \mathcal{R}^{\mathrm{WW}}[u(p)], \qquad (3)$$

so that

$$C^{\mathrm{WW}}(p_1, p_2, \ldots, p_n)$$
$$= \partial^n \mathcal{G}^{\mathrm{WW}}[u(p)]/\partial u(p_1) \ldots \partial u(p_n) \mid_{u=0} . \qquad (4)$$

Analogously, one can define the generating functionals for the final-state hadrons in two-jet WW decay,

$$\mathcal{R}^{\mathrm{W}}[u(p)] = 1 + \sum_{n=1}^{\infty} \frac{1}{n!} \int_{\Omega} \rho^{\mathrm{W}}(p_1, p_2, \ldots, p_n)$$

$$\times u(p_1) \ldots u(p_n) \prod_{i=1}^{n} \mathrm{d}p_i , \qquad (5)$$

$$\mathcal{G}^{\mathrm{W}}[u(p)] = \ln \mathcal{R}^{\mathrm{W}}[u(p)] \qquad (6)$$

with $\rho^{\mathrm{W}}(p_1, p_2, \ldots, p_n)$ being the n-particle inclusive density for two-jet WW decay.

Let us consider an uncorrelated WW decay scenario. In this we assume that each W boson showers and fragments into final-state hadrons without any reference to what is happening to the other. In this case $\mathcal{R}^{\mathrm{WW}}[u(p)]$ is the product of the generating functionals for the two-jet WW decay of differently charged W's

$$\mathcal{R}^{\mathrm{WW}}[u(p)] = \mathcal{R}^{\mathrm{W}^+}[u(p)] \, \mathcal{R}^{\mathrm{W}^-}[u(p)]. \qquad (7)$$

In terms of the generating functionals for the correlation functions, this can be represented as follows

$$\mathcal{G}^{\mathrm{WW}}[u(p)] = \mathcal{G}^{\mathrm{W}^+}[u(p)] + \mathcal{G}^{\mathrm{W}^-}[u(p)]. \qquad (8)$$

We explore the relation (7) only for the two-particle inclusive density. Being a simple characteristic beyond single-particle inclusive spectra, it is this quantity that is very often used in the correlation analysis, especially in connection with the BE interference.

2.2 Two-particle inclusive density

Let us first define the two-particle inclusive density $\rho(1,2)$ for particles 1 and 2 in the variable $Q_{12} = \sqrt{-(p_1 - p_2)^2}$ as

$$\rho(1,2) = \frac{1}{N_{\mathrm{ev}}} \frac{\mathrm{d}n_{\mathrm{pairs}}}{\mathrm{d}Q_{12}} ,$$

$$\int_Q \rho(1,2)\mathrm{d}Q_{12} = \langle n_1(n_2 - \delta_{12}) \rangle . \qquad (9)$$

Here, N_{ev} is the number of events (in a theoretical limit $N_{\mathrm{ev}} \to \infty$), n_{pairs} is the number of particle pairs, n_1 is the number of particles of type 1 in the event, n_2 that of type 2. For different hadrons (or identical hadrons coming from different events) $\delta_{12} = 0$ and $\delta_{12} = 1$ for identical hadrons coming from the same event. Since there are two possible combinations for identical hadrons, positive-positive and negative-negative, we combine both samples into a single one with a factor $1/2$. Hereafter, we shall refer to this as the like-charged particle sample and will symbolize it as $(\pm, \pm)$. For unlike-charged particle combinations, we adopt the notation $(+, -)$. The integration is performed in (9) over the full range of Q of the variable Q_{12}, so that

$$\langle n_1(n_2 - 1) \rangle \equiv F_2, \qquad (10)$$

where F_2 is the second-order (unnormalized) factorial moment for full phase space.

The single-particle and two-particle inclusive densities in Q_{12} variable for the four-jet WW hadronic decay can directly be obtained performing two successive functional differentiations of (7) over the probing function $u(p)$,

$$\rho^{\mathrm{WW}}(1) = \rho^{\mathrm{W}^+}(1) + \rho^{\mathrm{W}^-}(1) , \qquad (11)$$

$$\rho^{\mathrm{WW}}(1,2) = \rho^{\mathrm{W}^+}(1,2) + \rho^{\mathrm{W}^-}(1,2)$$
$$+ 2\rho^{\mathrm{W}^+}(1)\rho^{\mathrm{W}^-}(2). \qquad (12)$$

Note that the latter expression differs from the sum of two-particle densities for each independent source taken separately.

Performing the same functional differentiations of (8), one can find the two-particle correlation function in the four-jet WW decay,

$$C^{\mathrm{WW}}(1,2) = C^{\mathrm{W}^+}(1,2) + C^{\mathrm{W}^-}(1,2), \qquad (13)$$

This illustrates the fact that, in contrast to the two-particle densities, the correlation functions are additive and do not contain the contribution from lower-order inclusive densities.

Experimentally, it is advantageous to rewrite (12):

$$\rho^{\mathrm{WW}}(1,2) = \rho^{\mathrm{W}^+}(1,2) + \rho^{\mathrm{W}^-}(1,2) + 2\rho_{\mathrm{mix}}^{\mathrm{W}^+\mathrm{W}^-}(1,2), \quad (14)$$

where we replaced $\rho^{\mathrm{W}^+}(1)\rho^{\mathrm{W}^-}(2)$ with the track mixing two-particle density $\rho_{\mathrm{mix}}^{\mathrm{W}^+\mathrm{W}^-}(1,2)$ obtained by pairing particles from different two-jet WW events, to insure that particles coming from differently charged W's do not correlate. This technique leads to factorization of $\rho_{\mathrm{mix}}^{\mathrm{W}^+\mathrm{W}^-}(1,2)$ into the product of the single-particle densities.

Let us consider different charged-particle combinations. Following (14), one can define

$$\Delta\rho(\pm, \pm) \equiv \rho^{\mathrm{WW}}(\pm, \pm) - 2\,\rho^{\mathrm{W}}(\pm, \pm)$$
$$- 2\rho_{\mathrm{mix}}^{\mathrm{W}^+\mathrm{W}^-}(\pm, \pm) , \qquad (15)$$

$$\Delta\rho(+, -) \equiv \rho^{\mathrm{WW}}(+, -) - 2\,\rho^{\mathrm{W}}(+, -)$$
$$- 2\rho_{\mathrm{mix}}^{\mathrm{W}^+\mathrm{W}^-}(+, -) , \qquad (16)$$

where we assume that

$$\rho^{\mathrm{W}}(+, -) \equiv \rho^{\mathrm{W}^+}(+, -) = \rho^{\mathrm{W}^-}(+, -) , \qquad (17)$$

$$\rho^{\mathrm{W}}(\pm, \pm) \equiv \rho^{\mathrm{W}^+}(\pm, \pm) = \rho^{\mathrm{W}^-}(\pm, \pm) . \qquad (18)$$

Expressions (15) and (16) are evidently equal to zero for uncorrelated four-jet WW decay.

One can integrate (11) and (12) over the Q interval to obtain the relations for average multiplicity and second-order factorial moment in uncorrelated four-jet WW decays:

$$\Delta \equiv \langle n_{\mathrm{WW}} \rangle - \langle n_{\mathrm{W}^+} \rangle - \langle n_{\mathrm{W}^-} \rangle = 0 , \qquad (19)$$

$$\Delta F_2 \equiv F_2^{\mathrm{WW}} - F_2^{\mathrm{W}^+} - F_2^{\mathrm{W}^-} - 2\langle n_{\mathrm{W}^+} \rangle \langle n_{\mathrm{W}^-} \rangle = 0 . \quad (20)$$

The latter equation can also be directly obtained from the assumption on uncorrelated WW decay. Indeed, taking into account (10), (20) can be rewritten as

$$\langle n_{\mathrm{WW}}^2 \rangle - \langle n_{\mathrm{WW}} \rangle - \langle n_{\mathrm{W+}}^2 \rangle + \langle n_{\mathrm{W+}} \rangle - \langle n_{\mathrm{W-}}^2 \rangle + \langle n_{\mathrm{W-}} \rangle$$
$$-2 \langle n_{\mathrm{W+}} \rangle \langle n_{\mathrm{W-}} \rangle = 0 \ . \tag{21}$$

Assuming that for each WW event $n_{\mathrm{WW}} = n_{\mathrm{W+}} + n_{\mathrm{W-}}$ and, for uncorrelated WW decay, $\langle n_{\mathrm{W+}} n_{\mathrm{W-}} \rangle = \langle n_{\mathrm{W+}} \rangle \langle n_{\mathrm{W-}} \rangle$, one can see that the left-hand side of this equation is indeed zero. Note that in this particular case all $\langle n_{\mathrm{W}}^2 \rangle$ terms cancel, i.e., (21) holds for any full-phase-space multiplicity distribution.

By construction,

$$\Delta F_2 = \int_Q \Delta \rho(1,2), \tag{22}$$

omitting the charge dependence for simplicity.

A deviation of $\Delta \rho(\pm, \pm)$, $\Delta \rho(+, -)$ or ΔF_2 from zero is possible only in the case of correlated WW decay. It is very important to note, however, that the opposite is not true: $\Delta \rho(\pm, \pm) = \Delta \rho(+, -) = 0$ is a necessary, but not a *sufficient* condition for uncorrelated WW decay. This is further illustrated in the appendix using generating functions.

2.3 Reduced BE correlations in independent WW decay

A commonly acceptable method to study the BE effect is based on the calculation of the following correlation function:

$$R(1,2) = \frac{\rho(1,2)}{\rho(1)\rho(2)} = 1 + \frac{C(1,2)}{\rho(1)\rho(2)}. \tag{23}$$

In this it is assumed that the two-particle density $\rho(1,2)$ for identical (like-charged) boson combinations contains no additional correlations except those connected with the BE interference.

Experimentally, the reference sample $\rho(1)\rho(2)$ is usually constructed by using the track mixing method of pairing identical particles from different events. To make it possible to estimate the BE effect in the case when some extra correlations are present in $\rho(1,2)$, R should be further divided by the same function but calculated from Monte Carlo (MC) models without the BE effect. This technique is based on the assumption that different types of correlations can be factorized and Monte Carlo models are able to describe all other possible correlations correctly.

Another way to estimate R is to use the reference sample composed of unlike-charged particles from the same event. This method is affected by the presence of dynamical correlations due to the decay products of resonances.

A first attempt to describe the BE correlations in four-jet WW decay would be to understand the behavior of $C(1,2)$ when there is no stochastic dependence between W

pairs. This can be done if one remembers that the overall topology of $W^+W^- \to \bar{q}q l \bar{\nu}_l$ events is quite similar to that of Z boson decay at LEP1 energies. Therefore, one can assume that the correlation function $C^{\mathrm{W}}(1,2)$ in the two-jet WW events is the same as the correlation function $C^{\mathrm{Z}}(1,2)$ in Z boson decay. From (23) one can write

$$\rho^{\mathrm{W}}(1,2) = \rho^{\mathrm{W}}(1)\rho^{\mathrm{W}}(2) + C^{\mathrm{Z}}(1,2). \tag{24}$$

Substituting this into (12) assuming $\rho^{\mathrm{WW}}(1) = 2\rho^{\mathrm{W}}(1)$ *for full overlap in* Q_{12}, one has

$$\rho^{\mathrm{WW}}(1,2) = \rho^{\mathrm{WW}}(1)\rho^{\mathrm{WW}}(2) + 2\, C^{\mathrm{Z}}(1,2) \tag{25}$$

and

$$R^{\mathrm{WW}}(1,2) = 1 + \frac{1}{2}\frac{C^{\mathrm{Z}}(1,2)}{\rho^{\mathrm{W}}(1)\rho^{\mathrm{W}}(2)}. \tag{26}$$

From this follows the fact that, in the absence of WW correlations, the strength of the BE correlations in four-jet WW events is only half of the strength in Z boson or in two-jet W decay! In practice, the overlap will not be complete, even in the particular projection variable used, and the suppression will be less severe in actual Monte Carlo simulation below, but the point is that not the same, but a reduced BE effect has to be expected for WW events even in the absence of inter-W correlations. Note that the possibility of a decrease of the BE effect in the case of independent four-jet WW decay has already been pointed out in [8], but without quantitative estimates of this effect.

Of course, the latter conclusion is correct only in the case of no correlations between hadrons coming from different W bosons. We shall discuss the degree of validity of this assumption in the next subsection.

2.4 Monte Carlo study

To check the validity of $\Delta \rho(\pm, \pm) = \Delta \rho(+, -) = 0$ in (15) and (16), we use the PYTHIA 6.1 Monte Carlo model [11] with the L3 default parameters [12] for LUND hadronization without BE correlations[1]. A cut on charged-particle multiplicity $N_{\mathrm{ch}} > 2$ is used. The total number of events is 4000 for four-jet and 8000 for two-jet WW decays generated at c.m. energy of 190 GeV. Since the hadronic multiplicity of two-jet events is affected by τ decays, hadrons from τ decays are excluded. For the given statistics and tuning, the average charged-particle multiplicity is $\langle n_{\mathrm{W}} \rangle = 16.90 \pm 0.05$ for two-jet ($W^+W^- \to \bar{q}q l \bar{\nu}_l$) and $\langle n_{\mathrm{WW}} \rangle = 33.6 \pm 0.1$ for four-jet $W^+W^- \to \bar{q}_1\bar{q}_2 q_3 q_4$ decay. This is smaller than for the original JETSET default since long-lived resonances (such as K^0, Λ) are declared to be stable. From the mean multiplicities, one obtains the ratio $\langle n_{\mathrm{WW}} \rangle / 2 \langle n_{\mathrm{W}} \rangle = 0.994 \pm 0.004$.

To obtain $\rho_{\mathrm{mix}}^{\mathrm{W^+W^-}}$ in the track-mixing method, we generate the particle multiplicity N_{p} according to the Poisson

---

[1] We use the L3 default since in this paper we study the model with the two sets of parameters - with and without the BE simulation. Both models have been tuned to reproduce the same global-shape variables and single-particle densities at Z peak energy
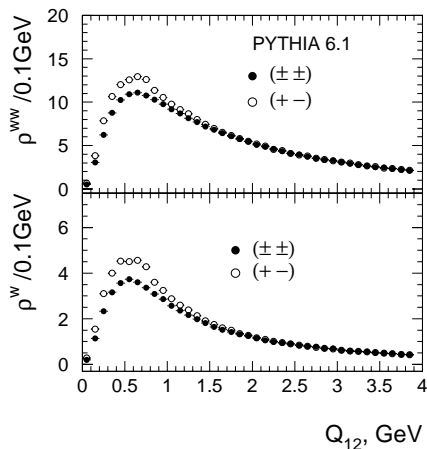
Fig. 1. Two-particle inclusive densities for four-jet and two-jet WW decays generated with PYTHIA MC without BE correlations
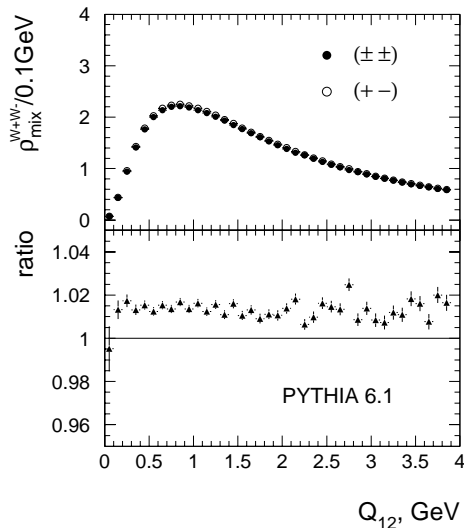

Fig. 2. Two-particle densities obtained with the track mixing method. Since it is difficult to distinguish between different charged combinations, the figure also shows the ratio $\rho_{\mathrm{mix}}^{\mathrm{w}^+\mathrm{w}^-}(\pm,\pm)/\rho_{\mathrm{mix}}^{\mathrm{w}^+\mathrm{w}^-}(+,-)$

distribution with the mean obtained from two-jet WW events. Then, we generate events using N_{p} tracks from two-jet WW events imposing the constraint that each track should originate from a different event. We require the total charge of the generated event to be zero, and that the two particles of the pair originate from differently charged W's. In addition, for a given generated event with the multiplicity N_{p}, only tracks from an original event of multiplicity $N_{\mathrm{p}} - 4 \leq N \leq N_{\mathrm{p}} + 4$ are used. The analysis is based on 250k track-mixed events.

Figures 1 and 2 show the behavior of the three terms in (15) and (16). Since the results for $\rho^{\mathrm{w}^+\mathrm{w}^-}(\pm,\pm)$ and $\rho_{\mathrm{mix}}^{\mathrm{w}^+\mathrm{w}^-}(+,-)$ are nearly on top of each other, Fig. 2 also shows the ratio $\rho_{\mathrm{mix}}^{\mathrm{w}^+\mathrm{w}^-}(\pm,\pm)/\rho_{\mathrm{mix}}^{\mathrm{w}^+\mathrm{w}^-}(+,-)$. Finally, Fig. 3 shows $\Delta\rho(\pm,\pm)$ and $\Delta\rho(+,-)$. As seen the assumption of independent hadronic W decay does not hold, even
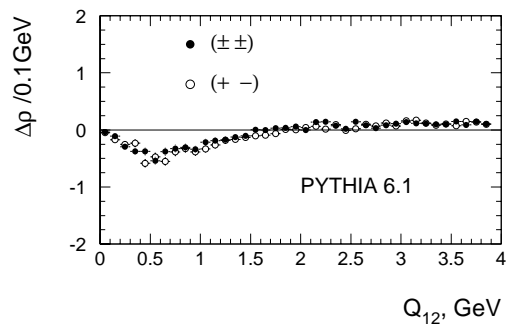

Fig. 3. $\Delta\rho$ obtained with PYTHIA MC without BE correlations

when color reconnection and BE effects are not included in the MC code. The degree of such non-independence can be estimated by integrating the left hand-side of (15) and (16) over full phase space to obtain ΔF_2 (20):

$$\int_Q \Delta\rho(\pm,\pm) \simeq \int_Q \Delta\rho(+,-) \simeq -4.3 \pm 0.3. \qquad (27)$$

Note that to simplify our study, we evaluated statistical errors assuming that there are no correlations between points at different Q_{12} values. This is a strong assumption, especially for the $Q_{12} \sim 0.5$ region where the contribution of resonances is largest and phase-space points are strongly correlated. In addition, we did not take into account that the average multiplicity $\langle n_{\mathrm{W}} \rangle$ has its error, which should be taken into account when generating the particle multiplicity N_{p} for $\rho_{\mathrm{mix}}^{\mathrm{W}^+\mathrm{W}^-}$ according to the Poisson distribution. Therefore, the values of errors shown in the figures are lower limits.

Statistical errors for $\rho_{\mathrm{mix}}^{\mathrm{w}^+\mathrm{w}^-}$ are rather small. Indeed, statistics available for the calculations of ρ^{W} is determined by the number of pairs. This is proportional to $\langle n \rangle_{\mathrm{W}}^2 N_{\mathrm{ev}}$, where N_{ev} is the number of two-jet events. However, if we do not take into account the cut $N_{\mathrm{p}} - 4 \leq N \leq N_{\mathrm{p}} + 4$, $\rho_{\mathrm{mix}}^{\mathrm{w}^+\mathrm{w}^-}$ is roughly determined by $\langle n \rangle_{\mathrm{W}}^2 N_{\mathrm{ev}}^2$ since tracks are taken from different events.

To check the correctness of the method, we simulated pseudo-W events using hadron production at the Z peak. The average multiplicity of these events is rather similar to that of two-jet WW decay. We combined two independent Z boson events generated with PYTHIA 6.1. Then, we considered this hypothetical event to be a "four-jet WW event". A single Z boson decay is considered to be a "two-jet WW event". Since Z boson decay products are taken from different events, they *a-priori* do not correlate and (15) and (16) have to be zero. Using the same program as that for the original WW sample, we repeated the previous calculations. The results are given in Fig. 4a.

To see whether there is any effect from the fact that the two W's in a four-jet event carry opposite charge, we repeated the above analysis combining two two-jet events with different W charge into a single "four-jet event". The result is given in Fig. 4b.

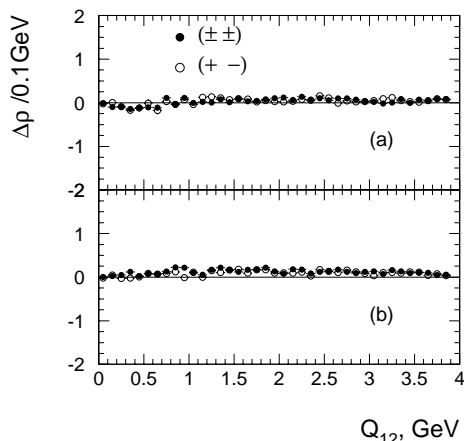In both figures, Fig. 4a and 4b, there is a small systematic deviation from the zero line. This can be due to

Fig. 4. $\Delta\rho$ obtained with PYTHIA MC without BE correlations, combining **a** two different Z boson events into one single four-jet event, **b** two two-jet events of opposite W charge into a single four-jet event

residual correlations which are not completely removed in the track-mixing sample. However, taking into account that statistical errors are underestimated, such a deviation is rather small and will be neglected.

3 Correlated WW production

3.1 General features

From the MC study in the previous section it follows that the standard assumption of independent WW decay is a rather naive simplification when we are dealing with the two-particle inclusive densities. One can consider a few possible reasons leading to non-independent WW decay:

1) Energy-momentum conservation. Consider the production of two W's in the c.m. frame. The mass of each W boson is distributed according to the Breit-Wigner shape, i.e. for each event the two masses are unequal and differ from the nominal W mass (which is 80.25 GeV for the L3 default in PYTHIA). From this it can be seen that there is a competition between the Breit-Wigner mass distribution and overall momentum rescaling to conserve the total energy E_{cm} and to allow for enough phase space.

2) Apart from the Breit-Wigner distribution, the overall topology of WW events is generated according to the matrix element approach with the nominal W mass. This calculation includes Coulomb interaction between different W's [11]. Theoretical calculations of the Coulomb effects on the WW production can be found in [13].

3) Since spin information is included into the matrix elements, there are angular correlations.

While the contribution from the two last effects is not well understood yet and, presumably, is small, the first effect is most important since it may produce negative correlations: The overall shape of the multiplicity distribution in the four-jet WW decay is slightly narrower than

that expected from naive superposition of two independent two-jet WW decays.

From the MC study, one can estimate the degree of (linear) stochastic dependence between two W masses. For this one can calculate the correlation coefficient,

$$r(m_+, m_-) = \left[\sigma^2(m_+)\sigma^2(m_-)\right]^{-1/2}$$
$$\times \left(\langle m_+ m_-\rangle - \langle m_+\rangle\langle m_-\rangle\right) , \quad (28)$$
$$-1 \leq r(m_+, m_-) \leq 1 , \quad (29)$$

where $\sigma^2(m_\pm)$ is the variance and $\langle\ldots\rangle$ stands for the average over all events. If there is no correlation between the two W masses m_+ and m_-, then $r(m_+, m_-) = 0$. Our MC estimate gives $r(m_+, m_-) \simeq -0.04 \pm 0.02$, i.e., there is a small negative correlation between masses. Since the multiplicity of particles is determined by m_+ and m_-, this means that a large hadron multiplicity from one W boson slightly suppresses the multiplicity of hadrons coming from the other W.

The effects discussed above are not the only phenomena which can lead to non-zero values of (15) and (16). At hadronization scale distances, the space separation between the two W decay vertices is rather small (~ 0.1 fm) and the hadronization regions of the two W bosons overlap. For this system, soft partons originating from different W bosons are close-by in space and could form color-singlet clusters from which the observable final-state hadrons emerge [14]. Therefore, the origin of these hadrons is difficult to determine. Such an effect, usually called color reconnection, could lead to an additional non-independency of W decay products. In terms of the LUND model, the reconnection occurs when strings overlap like for a type I superconductor or when they cross like for a type II superconductor [8].

After the transformation of partons into hadrons, the BE correlations can give an additional contribution to the overall correlations, since the space-time separation between hadrons is still smaller than the typical source radii ($\sim 0.5 - 1.0$ fm) of the BE correlations.

In the case of the interference effects, one can assume

$$\Delta\rho(\pm,\pm) = \Delta\rho^{ec}(\pm,\pm) + \Delta\rho^{be}(\pm,\pm)$$
$$+ \Delta\rho^{cr}(\pm,\pm) , \quad (30)$$
$$\Delta\rho(+,-) = \Delta\rho^{ec}(+,-) + \Delta\rho^{cr}(+,-) , \quad (31)$$

where $\Delta\rho^{ec}$ is the contribution from energy conservation and other non-interference effects, $\Delta\rho^{be}$ represents the BE correlations and $\Delta\rho^{cr}$-color-reconnection correlations.

One can directly investigate the interference effects by calculating the difference:

$$\delta\rho = \Delta\rho(\pm,\pm) - \Delta\rho(+,-). \quad (32)$$

Since the track mixing terms are very similar, one has

$$\delta\rho \simeq \rho^{ww}(\pm,\pm) - 2\rho^{w}(\pm,\pm)$$
$$- \rho^{ww}(+,-) + 2\rho^{w}(+,-), \quad (33)$$

which no longer involves the track mixing terms since they cancel. Taking into account the fact that $\Delta\rho^{ec}(\pm,\pm)$ and

$\Delta\rho^{\mathrm{ec}}(+,-)$ are the same (see MC studies above), from (30) and (31) one can see that $\delta\rho$ resolves only the interference terms

$$\delta\rho \simeq \Delta\rho^{\mathrm{be}}(\pm,\pm) + \Delta\rho^{\mathrm{cr}}(\pm,\pm) - \Delta\rho^{\mathrm{cr}}(+,-). \quad (34)$$

If the color-reconnection effects are charge-independent, $\delta\rho$ is fully determined by the BE correlations, $\delta\rho \simeq \Delta\rho^{\mathrm{be}}(\pm,\pm)$.

3.2 BE correlations

The study of BE interference in the form of an enhancement of the two-particle correlation function by comparing fully hadronic and double semi-leptonic events has been proposed in [1]. Following this method, the DELPHI Collaboration measured the following correlation function [4]:

$$R^* = \frac{\rho^{\mathrm{WW}}(\pm,\pm) - 2\,\rho^{\mathrm{W}}(\pm,\pm)}{\rho^{\mathrm{WW}}(+,-) - 2\,\rho^{\mathrm{W}}(+,-)}. \quad (35)$$

Because of (15) and (16), this expression is equal to

$$R^* = \frac{2\rho_{\mathrm{mix}}^{\mathrm{W^+W^-}}(\pm,\pm) + \Delta\rho(\pm,\pm)}{2\rho_{\mathrm{mix}}^{\mathrm{W^+W^-}}(+,-) + \Delta\rho(+,-)}. \quad (36)$$

Note that (36) has very little to do with the standard definition of the BE correlation function (23).

This can be seen if one assumes that $\Delta\rho(+,-) = 0$ and $\rho_{\mathrm{mix}}^{\mathrm{W^+W^-}}(\pm,\pm) \simeq \rho_{\mathrm{mix}}^{\mathrm{W^+W^-}}(+,-)$,

$$R^* \sim 1 + \frac{\Delta\rho(\pm,\pm)}{2\rho_{\mathrm{mix}}^{\mathrm{W^+W^-}}(\pm,\pm)}. \quad (37)$$

Formally, the structure of (37) is similar to (23). However, since $\Delta\rho(\pm,\pm)$ is different from $C(1,2)$ for identical pions originating from different W bosons, R^* is not the BE correlation function. For example, one can see that R^* is always peaked at $Q_{12} \to 0$ for any slow change in $\Delta\rho(\pm,\pm)$, since $\rho_{\mathrm{mix}}^{\mathrm{W^-W^+}}(\pm,\pm)$ is a decreasing function for $Q_{12} \to 0$. In fact, $\Delta\rho(\pm,\pm)$ is non-dynamically distorted by this division. Such a distortion by the single-particle density in (35)-(37) is properly removed in the definition (33) to study the interference effects.

3.3 Monte Carlo studies

To see how the BE correlations affect $\delta\rho$ and R^*, we use the PYTHIA 6.1 Monte Carlo with the BE effect included for all identical pions. The BE correlations are simulated with the LUBOEI model. After the model retuning, the average charged-particle multiplicity is $\langle n_{\mathrm{W}} \rangle = 16.72 \pm 0.05$ for two-jet and $\langle n_{\mathrm{WW}} \rangle = 33.5 \pm 0.1$ for four-jet WW decay. The ratio is $\langle n_{\mathrm{WW}} \rangle / 2\langle n_{\mathrm{W}} \rangle = 0.997 \pm 0.004$.

Figures 5, 6, 7 show the terms of (15) and (16) for the case of the BE correlations, as they are implemented into the Monte Carlo code. The most obvious difference is the (expected) effect of the BE correlations on $\Delta\rho(\pm,\pm)$ at
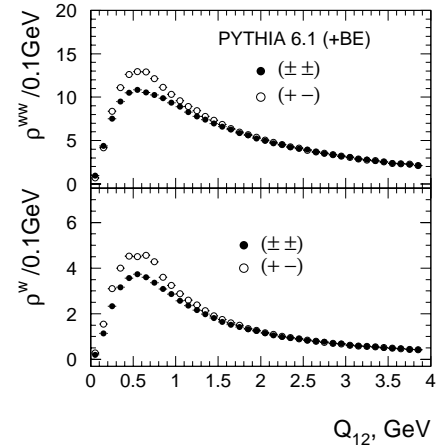

Fig. 5. Two-particle inclusive densities for fully hadronic and semi-leptonic WW decays generated with PYTHIA MC with BE correlations included
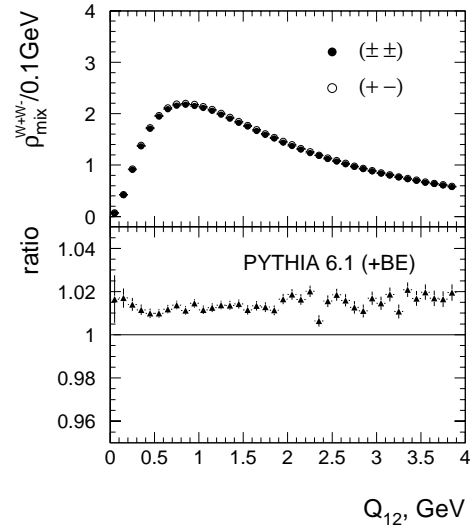

Fig. 6. Two-particle densities obtained with the track mixing method for PYTHIA with BE correlations. The figure also shows the ratio $\rho_{\mathrm{mix}}^{\mathrm{WW}}(\pm,\pm)/\rho_{\mathrm{mix}}^{\mathrm{WW}}(+,-)$
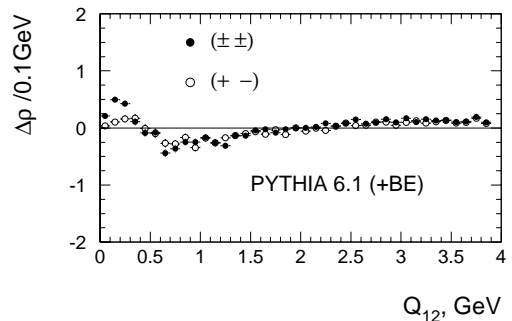

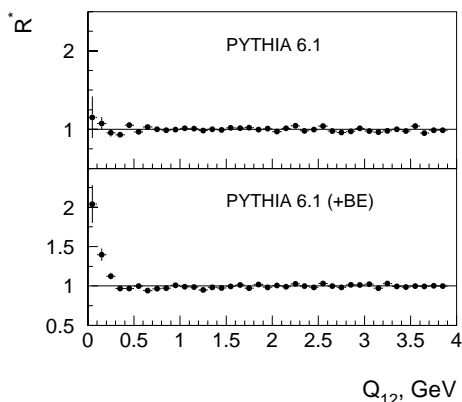**Fig. 7.** $\Delta\rho$ for PYTHIA MC with BE correlations

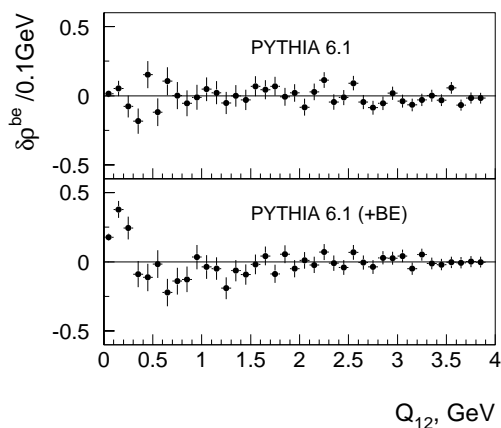**Fig. 8.** R^* for PYTHIA MC without and with BE correlations

Fig. 9. $\delta\rho$ for PYTHIA MC without and with BE correlations

small Q_{12} in Fig. 7 (c.f. Fig. 3). However, also $\Delta\rho(+,-)$ is affected and non-zero in LUBOEI. The approximation (37), therefore, is not valid and (36) cannot measure the standard BE correlation function. Integrating the left hand-side of (15) and (16) over full phase space, one has (c.f. (27))

$$\int_Q \Delta\rho(\pm,\pm) \simeq 1.54 \pm 0.04 \,,$$
$$\int_Q \Delta\rho(+,-) \simeq 1.43 \pm 0.04 \,. \tag{38}$$

Figures 8 and 9 show the behavior of R^* and $\delta\rho$. The BE effect appears stronger in R^* than in $\delta\rho$. In addition, statistical errors in Fig. 8 are much smaller. However, as we have noted already, this is mainly because of the form of $\rho_{\mathrm{mix}}^{\mathrm{WW}}(\pm,\pm)$ at small Q_{12}. This leads to a behavior of R^* appearing similar to that of BE correlations, even if $\Delta\rho(\pm,\pm)$ is a small Q_{12}-independent constant.

The inconsistency in the BE correlation study by means of R^* can be seen in Fig. 8. In the parameterization of the BE correlations, the L3 default is a spherical Gaussian source $R(Q_{12}) \sim 1 + \lambda\exp(-r^2 Q_{12}^2)$ with the correlation strength parameter $\lambda = 1.5$ and radius $r = 0.6$ for all 9 particle species. This means that the BE from

different W bosons should have a similar form. However, Fig. 8 shows that the shape is far from Gaussian.

The structure of the BE correlations between hadrons originating from different W bosons can be observed from the study of $\delta\rho$, despite its evidently small signal. One can see from Fig. 7 that LUBOEI changes the unlike-particle spectrum as well: Since LUBOEI spoils the overall energy-momentum conservation when it shifts identical particles to reproduce the expected two-particle correlation function, momenta of non-identical particles are modified. Assuming that there is no color reconnection, expressions (30), (31) and (34) are modified for LUBOEI as

$$\Delta\rho(\pm,\pm) = \Delta\rho^{\mathrm{ec}}(\pm,\pm) + \Delta\rho_{\mathrm{LUB}}^{\mathrm{be}}(\pm,\pm) \,, \tag{39}$$
$$\Delta\rho(+,-) = \Delta\rho^{\mathrm{ec}}(+,-) + \Delta\rho_{\mathrm{LUB}}^{\mathrm{be}}(+,-) \,, \tag{40}$$
$$\delta\rho = \Delta\rho(\pm,\pm) - \Delta\rho(+,-) = \Delta\rho_{\mathrm{LUB}}^{\mathrm{be}}(\pm,\pm)$$
$$- \Delta\rho_{\mathrm{LUB}}^{\mathrm{be}}(+,-) \,, \tag{41}$$

where $\Delta\rho_{\mathrm{LUB}}^{\mathrm{be}}(\pm,\pm)$ and $\Delta\rho_{\mathrm{LUB}}^{\mathrm{be}}(+,-)$ are the terms due to the BE interference simulated with LUBOEI. As can be seen, $\delta\rho$ resolves the comparatively large difference between $\Delta\rho_{\mathrm{LUB}}^{\mathrm{be}}(\pm,\pm)$ and $\Delta\rho_{\mathrm{LUB}}^{\mathrm{be}}(+,-)$, rather than the distortion for like-charged particles alone. The effect depends on the amount of change in the unlike-charged particle spectra and other implementations of the BE interference may show a different effect for $\delta\rho$ than that observed from the LUBOEI model. (In the limit that the BE interference would not change the unlike-charged spectra, the signal would be as much as two to three times stronger for $\delta\rho$ than that observed from the LUBOEI model.) It would be interesting to apply other BE simulations based, for example, on local [15] and global [16] re-weighting methods or on the LUND string model [17,18].

Note that the color reconnection effect cannot be detected using (32) if it is charge independent, unlike to the BE correlations. However, the color reconnection can be detected from unlike-charged particle combinations, after properly removing the correlations from energy-conservation.

4 Conclusion

One of the main reasons to study the BE and color reconnection effects is the possibility to determine the precision with which the W mass can be measured at LEP2 energies. Moreover, such investigations provide an opportunity for probing the structure of the QCD vacuum and the details of hadronization.

In this paper we discuss model-independent signatures of the BE and color reconnection effects beyond single-particle spectra. The problem of the two-particle correlations in the WW system, however, is not as simple as it looks at a first glance: for WW events without color reconnection or BE correlations, there are energy-momentum and other correlations which can distort the observed two-particle densities. These correlations should be properly

taken into account before estimating the interference effects. We propose to calculate the difference $\delta\rho$, in which contributions from energy-conservation cancel. This differs from the method used by DELPHI. Formally, the latter resembles the traditional way of the BE correlation study, but any quantitative interpretation of strength and radius parameters and comparison to values obtained from previous BE analysis is misleading. The method proposed here is expected to be less sensitive to the distortion from other dynamical correlations between W's.

As a final remark, we note that even if the energy-conservation effects are properly removed as in (32), the absence of a signal is not a sufficient condition for the absence of interference at the hadronization scale. The correlations between hadrons originating from different W's may well exhibit themselves in higher-order inclusive distributions.

Appendix

Recently it was suggested [6] that the color reconnection effect can lead a smaller mean hadronic multiplicity $\langle n_{\mathrm{WW}}\rangle$ in fully hadronic decay than twice the hadronic multiplicity $\langle n_{\mathrm{W}}\rangle$ in semi-leptonic decay, i.e. Δ in (19) is negative. At the present level of statistics at LEP2, no such an effect has been found [5,19]. A Monte Carlo simulation of BE correlations based on the Lund Fragmentation Model gives no support to the possible experimental signal involving single-particle spectra as well [18,20].

Statistically, of course, (19) is not a condition for stochastic dependence between two systems. Purely independent production of W bosons has to lead to the factorization of the generating functionals as in (7). This can be illustrated by replacing auxiliary function $u(p)$ by a constant z. Then the generating functional is reduced to the generating function $G^{\mathrm{WW}}(z) = \sum(1+z)^n P_n$ for the probabilities P_n of detecting n hadrons in fully hadronic four-jet WW decays

$$G^{\mathrm{WW}}(z) = G^{\mathrm{W}^+}(z)\, G^{\mathrm{W}^-}(z) \;, \qquad (42)$$

$$F_q^{\mathrm{WW}} = \frac{\partial^q G^{\mathrm{WW}}(z)}{\partial z^q}\,|_{z=0} \;, \qquad (43)$$

where $G^{\mathrm{W}}(z)$ is the generating function for final-state particles in the two-jet WW decays and F_q^{WW} is the factorial moment.

If there is a stochastic dependence between W's, (42) has to be modified. However, rigorous information about the interdependence is necessary to make any definite statement about the exact form of $G^{\mathrm{WW}}(z)$. This information is not available because of many unknown factors. One may assume that $G^{\mathrm{WW}}(z)$ can still be represented by $G^{\mathrm{W}^+}(z)$

and $G^{\mathrm{W}^-}(z)$ if the distortions caused by such a dependence are not very strong. Then,

$$G^{\mathrm{WW}}(z) = G^{\mathrm{W}^+}(z)G^{\mathrm{W}^-}(z) + g(z), \qquad (44)$$

where $g(z)$ is a function representing possible stochastic dependence between decay products of different W's. To preserve the total normalization $G^{\mathrm{WW}}(z=-1) = 1$, one should put $g(z=-1) = 0$, so that $g(z)$ is not a generating function in the "usual" probabilistic sense. In addition, one must require $g'(z)\,|_{z=0}= 0$ and that the form of $g(z)$ cannot lead to $P_n < 0$. Such a method was used in [21] to introduce a stochastic dependence between Bernoulli sources in order to modify a positive-binomial distribution.

It is easy to see that $g(z)$ contains integrated properties of interference and other effects leading to the dependence of different W's. For the average multiplicity $\langle n_{\mathrm{WW}}\rangle = F_1^{\mathrm{WW}}$ in four-jet WW decay, one has from (44)

$$\langle n_{\mathrm{WW}}\rangle = \langle n_{\mathrm{W}^+}\rangle + \langle n_{\mathrm{W}^-}\rangle \qquad (45)$$

since $g'(z)\,|_{z=0}= \Delta = 0$.

For the second-order factorial moment, one obtains

$$F_2^{\mathrm{WW}} = F_2^{\mathrm{W}^+} + F_2^{\mathrm{W}^-} + 2\,\langle n_{\mathrm{W}^+}\rangle\,\langle n_{\mathrm{W}^-}\rangle + g''(z)\,|_{z=0} \;. \quad (46)$$

By comparing this expression with (20), one can see that

$$\Delta F_2 = g''(z)\,|_{z=0} \;. \qquad (47)$$

If it happens that $g''(z)\,|_{z=0}= 0$, then we shall not be able to detect the BE correlations and color reconnection. If this is so, higher-order factorial moments (or inclusive densities) would have to be checked before one is able to exclude interference effects.

References

1. L. Lönnblad, T. Sjöstrand, Phys. Lett. B **351**, 293 (1995)
2. Z. Kunszt et al., in "Physics at LEP2", eds. G. Altarelli, T. Sjöstrand, F. Zwirner CERN Report 96-01, Vol.1, p.141 (1996)
3. V.A.Khoze, T. Sjöstrand, CERN-TH/98-74
4. DELPHI Coll., P. Abreu et al., Phys. Lett. B **401**, 181 (1997)
5. DELPHI Coll., P. Abreu et al., Phys. Lett. B **416**, 233 (1998)
6. J. Ellis, K. Geiger, Phys. Rev. D **54**, 1155 (1996); Phys. Lett. B **404**, 230 (1997)
7. G. Gustafson, U. Peterson, P. Zerwas, Phys. Lett. B **209**, 90 (1988); G. Gustafson, J. Häkkinen, Z. Phys. C **64**, 659 (1994)
8. T. Sjöstrand, V.A. Khoze, Phys. Rev. Lett. **72**, 28 (1994); Z. Phys. C **62**, 281 (1994)
9. Š. Todorova-Nová, "Color Reconnection in String Model", DELPHI note (public) 96-158 PHYS 651
10. E.A.De Wolf, I.M. Dremin, W. Kittel, Phys. Reports **270**, 1 (1996)

11. T. Sjöstrand, Comp. Phys. Comm. **82**, 74 (1994); The supersymmetry extensions are described in S. Mrenna, Comp. Phys. Comm. **101**, 232 (1997)

12. J. Casaus, L3 Note 1946 (1996); Sunanda Banerjee and Swagato Banerjee, L3 Note 1978 (1996); I.G. Knowles, T. Sjöstrand (conveners), "QCD Event Generators", Physics at LEP2, CERN-96-01, Vol. 2, p.103 (1996)

13. V.S. Fadin, V.A. Khoze, A.D. Martin, A. Chapovsky, Phys. Rev. D **52**, 1377 (1995); V.A. Khoze, W.J. Stirling, Phys. Lett. B **356**, 373 (1995)

14. B.R. Webber, Presented at Phenomenology Workshop on LEP2 Physics, Oxford, April 1997, hep-ph/9708463

15. L. Lönnblad, T. Sjöstrand, Eur. Phys. J C **2**, 165 (1998)

16. S. Jadach, K. Zalewski, Acta Phys. Pol. B **28**, 1363 (1997); K. Fiałkowski, R. Wit, Z. Phys. C **74**, 145 (1997)

17. B. Andersson, Proc. 7th Int. Workshop on Multiparticle Production "Correlations and Fluctuations", Nijmegen, The Netherlands 1996, Eds: R.C. Hwa et al., (World Scientific, Singapore, 1997) p.86; B. Andersson, M. Ringnér, Nucl. Phys. B **513**, 627 (1998)

18. Š. Todorova-Nová, J. Rameš, "Simulation of Bose-Einstein Effect Using Space-Time Aspects of Lund String Fragmentation Model", hep-ph/9710280

19. D.R. Ward, *Talk given at the EPS Jerusalem Conference* (1997); L3 Coll., EPS-HER/97-814, L3 Note 2134 (1997); OPAL Coll., CERN-PPE/97-116 (1997); DELPHI Coll., EPS-HER/97-307 (1997)

20. J. Häkkinen, M. Ringnér, Eur. Phys. J C **5**, 275 (1998)

21. S.V. Chekanov, Acta Phys. Pol. B **25**, 1583 (1994)